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# Heat transfer in MHD flow with pressure gradient, suction and injection

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SUMMARY

Numerical solutions to the MHD Falkner-Skan equation and the corresponding heat transfer equation have been obtained on taking into consideration the effects of suction and injection and the pressure gradient parameter $\bar{\beta}$. Velocity and temperature profiles are shown on graphs and the numerical values of the skin friction and the rate of heat transfer are given in the form of tables. It has been observed that an increase in $N_m$ (magnetic field parameter) leads to an increase in velocity, skin friction, rate of heat transfer and a fall in temperature. Also an increase in suction leads to a fall in the value of the skin friction and the rate of heat transfer, opposite to the case of injection.

## 1. Introduction

MHD flows have been receiving considerable attention because of their practical applications. Many papers have been published on this topic, notable amongst them are by Hartmann [1], Hartmann and Freimut [2], Rossow [3], Shercliff [4], Lykoudis [5] dealing with various phenomena in MHD boundary layers. Pai [6], Sutton and Sherman [7], Hughes and Young [8] have also published text-books on this topic. Recently, Cobble [9] showed the conditions under which a similarity solution exists to MHD flow over a semi-infinite flat plate in the presence of a magnetic field and a pressure gradient with or without suction and injection. However, Cobble solved numerically the similarity equation for only one case, viz., $m = 0.1$, $N_m = 0.01$ and without suction and injection. Here Cobble defines the free-stream as $U_\infty = U_0 \ell^m$ $(\ell > 0)$ and $N_m$ is the magnetic field strength number. In the non-magnetic case, such a problem has been treated by many authors like Hartnett and Eckert [10] and Schlichting [11]. Moreover, the heat transfer aspect of such a MHD problem as discussed by Cobble has not been studied in the literature. Hence, it is now proposed to study the velocity field for different values of $\bar{\beta} = 2m/(m+1)$, $N_m$ and $f_w$, the latter being a suction/injection parameter, and the corresponding heat transfer aspect of the problem. In Sec. 2, the mathematical analysis has been presented and in Sec. 3, the conclusions have been set out.

## 2. Mathematical analysis

We consider the flow of an incompressible viscous electrically conducting fluid past a semi-infinite porous plate under the influence of a transversely applied magnetic field. Neglecting the

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induced magnetic field, the steady flow has been shown to be governed by the following equations (Ref. [9]):

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{g}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{g\sigma B_y^2(x)u}{\rho} \tag{1}$$

and

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{2}$$

Here $u$, $v$ are the components of the velocity in the $x$- and $y$-directions respectively, $g$ the acceleration due to gravity, $\rho$ the fluid density, $p$ the pressure, $\nu$ the kinematic viscosity, $\sigma$ electrical conductivity and $B_y$ is the magnetic field strength. With the usual similarity analysis, Cobble has shown that these equations reduce to the following ordinary differential equation with boundary conditions:

$$f'''+ff''+\bar{\beta}(1-f'^2)+N_m(1-f')=0,$$

$$f(0)=-v_0/b\beta=-f_w,\quad f'(0)=0\quad\text{and}\quad\lim_{\eta\to\infty}f'=1, \tag{3}$$

$$\text{where}\quad\bar{\beta}=2m/(m+1)\quad\text{and}\quad N_m=S_0/\beta U_0,\quad\beta=(1+m)/2.$$

As a numerical example, Cobble has given the numerical solution of (3) for $f(0)=0, f'(0)=0$, $m=0.1$ and $N_m=0.01$. As this problem is important from a technological point of view, additional numerical solutions of equations (3) are most useful. Hence, we have solved equation (3) numerically for more values of $\bar{\beta}$, $N_m$ and $f_w$, where $f_w$ is the suction/injection parameter.

Again the heat transfer aspect of this problem has not been studied in the literature. So we have presented the solution of the energy equation as follows.

The energy equation is given by

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

and the boundary conditions are

$$T=T_w\quad\text{at}\quad y=0,\quad T=T_\infty\quad\text{as}\quad y\to\infty. \tag{5}$$

Here $T$ is the temperature of the fluid, $T_w$ the plate temperature, $T_\infty$ the temperature of the fluid in the free stream, $\rho$ the density, $\kappa$ the thermal conductivity and $c_p$ is the specific heat at constant pressure.

Introducing the following transformations

$$u=abx^{\beta-\alpha}f'(\eta),\quad v=-bx^{\beta-1}(\beta f(\eta)-\alpha\eta f'(\eta)),$$

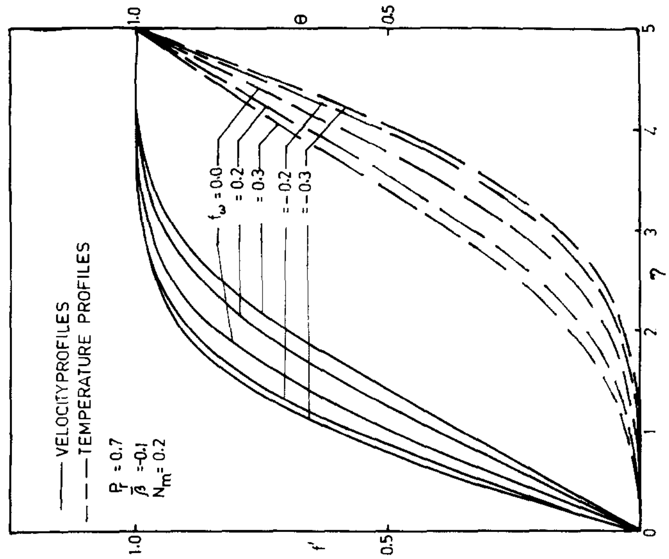


Fig. 3. Velocity and temperature profiles.

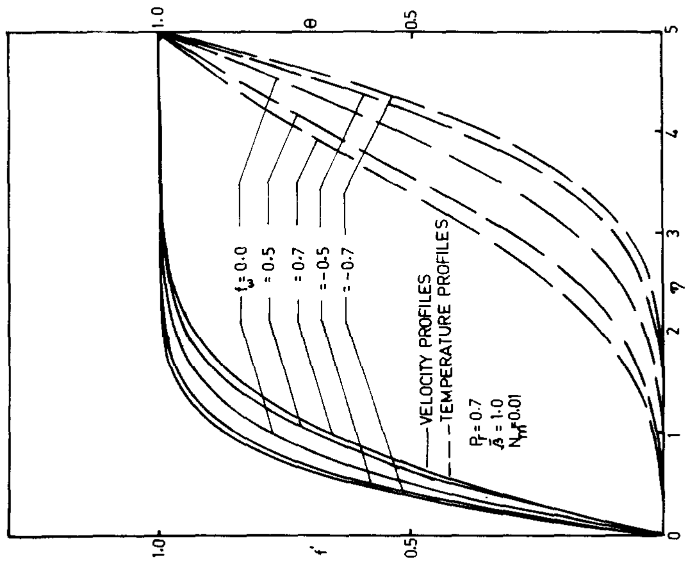


Fig. 2. Velocity and temperature profiles.

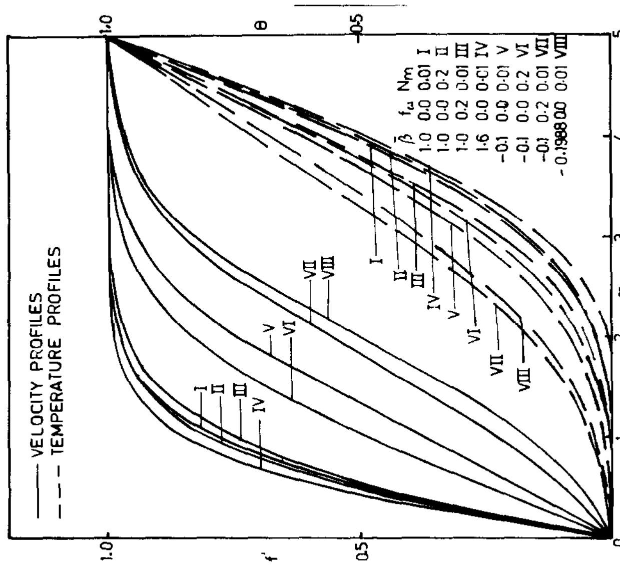


Fig. 1. Velocity and temperature profiles $P_r = 0.7$.

$$\eta = ay/x^{\alpha}, \quad \theta = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \quad a = \sqrt{\beta U_0/\nu}, \quad \beta = \frac{1+m}{2} \ (\beta \neq 0),$$

$$b = \sqrt{\nu U_0/\beta}, \quad \alpha = \frac{1-m}{2} \quad (\alpha > 0 \text{ for uniqueness of } \eta) \tag{6}$$

in equations (4) and (5), we get

$$\theta'' + Prf\theta' = 0, \quad Pr = \rho\nu c_p/\kappa$$

$$\theta(0) = 1, \quad \theta(\infty) = 0, \tag{7}$$

where primes now denote derivatives with respect to $\eta$. Numerical solutions to equations (7) have been derived for different values of $\bar{\beta}$, $N_m$ and $f_w$ for Prandtl number $Pr = 0.7$. The velocity and temperature profiles are shown in Figs. 1-3. In Fig. 1 they are shown for different values of $\bar{\beta}$, $f_w$ and $N_m$. We observe from this figure that an increase in $N_m$ leads to an increase

TABLE 1

Values of skin-friction and rate of heat transfer

| $Pr = 0.7$ | | | | |
|---|---|---|---|---|
| $\bar{\beta}$ | $f_w$ | $N_m$ | $f''(0)$ | $-\theta'(0)$ |
| 1.0 | 0.0 | 0.01 | 1.23661 | 0.49619 |
| 1.0 | 0.0 | 0.2 | 1.31061 | 0.50072 |
| 1.0 | 0.2 | 0.01 | 1.12549 | 0.40938 |
| 1.6 | 0.0 | 0.01 | 1.52483 | 0.52974 |
| −0.1 | 0.0 | 0.01 | 0.33262 | 0.38943 |
| −0.1 | 0.0 | 0.2 | 0.53199 | 0.42106 |
| −0.1 | 0.2 | 0.01 | 0.16245 | 0.26977 |
| −0.1988 | 0.0 | 0.01 | 0.07999 | 0.32558 |

| $Pr = 0.7$, $\bar{\beta} = 1.0$, $N_m = 0.01$ | | |
|---|---|---|
| $f_w$ | $f''(0)$ | $-\theta'(0)$ |
| 0.0 | 1.23661 | 0.49619 |
| 0.5 | 0.97340 | 0.29392 |
| 0.7 | 0.88215 | 0.22790 |
| −0.5 | 1.54550 | 0.74120 |
| −0.7 | 1.68024 | 0.84825 |

| $Pr = 0.7$, $\bar{\beta} = -0.1$, $N_m = 0.2$ | | |
|---|---|---|
| $f_w$ | $f'(0)$ | $-\theta'(0)$ |
| 0.0 | 0.53199 | 0.42106 |
| 0.2 | 0.40419 | 0.32163 |
| 0.3 | 0.34639 | 0.27543 |
| −0.2 | 0.67259 | 0.52616 |
| −0.3 | 0.74677 | 0.58052 |

in the value of the velocity when $f_w = 0$ and $\overline{\beta} = -0.1$, 1 whereas temperature, under these conditions, decreases. In the presence of injection, there is a rise in velocity and a fall in temperature. An increase in $\overline{\beta}$ ($> 0$) leads to a rise in velocity and a fall in temperature whereas an increase in $\overline{\beta}$ ($< 0$) leads to a fall in velocity and a rise in temperature. In Figs. 2 and 3, the effects of suction and injection have been shown on the velocity and temperature fields for $\overline{\beta} = -0.1, 1$. Here $f_w < 0$ indicates injection and $f_w > 0$ indicates suction. An increase in injection rate leads to a rise in velocity and a fall in temperature for both $\overline{\beta} = -0.1, 1$ whereas an increase in suction leads to a fall in velocity and a rise in temperature. Cobble has not considered the effects of $\overline{\beta}$, $N_m$ and $f_w$ on the skin friction and rate of heat transfer which are respectively proportional to $f''(0)$ and $\{-\theta'(0)\}$. We have calculated the numerical values of $f''(0)$ and $\{-\theta'(0)\}$ and these have been entered in Table I, for $Pr = 0.7$. We observe from this table that an increase in $\overline{\beta}$ leads to a rise in the skin friction and rate of heat transfer in the MHD case. Also, for $\overline{\beta} \lesseqgtr 0$, an increase in suction leads to a fall in the value of the skin friction and the rate of heat transfer, whereas an increase in the rate of injection leads to a rise in the skin friction and the rate of heat transfer.

## 3. Conclusions

1. An increase in $N_m$ leads to an increase in velocity, skin friction and rate of heat transfer and a fall in temperature.
2. An increase in $\overline{\beta}$ ($\gtreqless 0$) leads to a rise in the skin friction and the rate of heat transfer.
3. An increase in suction leads to a fall in the value of skin friction and the rate of heat transfer, opposite to the case of injection.

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